A PERFORMANCE Attribution Model for FIXED-INCOME PORTFOLIOS

Eleven factors to consider when evaluating bond holdings.

BY NABIL KHOURY, MARC VEILLEUX & ROBERT VIAU

Performance attribution analysis partitions a portfolio's *ex-post* return into specific components associated with particular decisions made during the management process, in order to assess their impact on overall performance. In the case of equity portfolios, attribution analysis typically breaks down *ex-post* return into three components related to asset allocation decisions, industry choice decisions and security selection decisions. The contribution of each component to the overall performance of the portfolio is then determined by calculating the negative or positive departure of its associated return from that of a corresponding benchmark. This process can shed light on the efficiency of the portfolio's management process and identify areas where changes could enhance performance.

Generally speaking, the *ex-post* holding period return of a bond portfolio can similarly be broken down into four broad components related to (I) passage of time; (II) yield curve movements (encompassing parallel shifts, changes in curvature and twists); (III) changes in spreads of various categories of bonds; and (IV) security selection. The model presented in this paper adopts this four-way subdivision of the *ex-post* return of fixed-income portfolios and differentiates in sufficient detail between the various areas of management expertise within each subdivision in order to give a clear breakdown of value added. Another important feature is the model's multiplicative approach which corresponds to the multiperiod evaluation that portfolio managers and investors are interested in. Indeed, most existing models of performance attribution for fixed-income portfolios rely on an additive approach to sum up the various sources of added value. They may work well when analyzing a single period (such as a month) but fail when performance attribution is done over several consecutive periods (six months, for example). Finally, the subdivision factors are then compared to benchmark returns in order to determine relative value added.

A factor model of bond portfolio holding period returns

In this study, we break down the *ex-post* holding period return of a typical bond portfolio into 11 factors. These factors represent a more detailed subdivision of the four components of realized return mentioned above. For each factor we estimate the periodic return attributable; in general, this return is calculated as the change in the market value of the portfolio due to this factor divided by the market value of the portfolio at the start of the period.

Passage of time

Factor 1 – The Time Factor: Portfolio discount or premium amortized during period *t*, calculated using the yield to maturity in the federal bond market[1] for securities of the same category and maturity as those in the portfolio.

The impact of this amortization on portfolio total return is calculated by subtracting the estimated market value of the portfolio at the beginning of

Nabil Khoury is professor emeritus at Université Laval's department of finance and insurance. Marc Veilleux is a portfolio manager at Natcan Investment Management, in Montreal and Robert Viau is director, financial training and client services at CDP du Québec in Montreal.

period *t* from its estimated market value at the end of that period (using in both cases the yields observed in the federal bond market as of the beginning of period *t* for bonds of the same category and maturity as those in the portfolio). This factor accounts for the convergence of the principal amount towards its nominal value. As for all factor impacts, this change is divided by the market value of the portfolio at the start of the period.

Factor 2 – The Coupon Factor: Coupons earned by the bond portfolio during period *t*.

Coupons earned may indeed account for a large fraction of a bond portfolio's total return. The change in value attributable to this factor is calculated by identifying the coupons received, plus the net variation in the accrued interest, resulting from purchases and liquidations that took place during period *t*.[2]

Factor 3 – The Cash Factor: Interest earned (foregone) on the money market position during period *t*.

Since coupons were included in the calculation of the previous factor, the interest earned (foregone) on the money market position during period *t* is estimated on the assumption that all money market securities are discount bonds.

We treat interest earned (foregone) on money market positions as a separate factor rather than including it as a class of bonds, since it is usually a buffer class used primarily to build up funds between transactions on bonds with more distant maturities.[3] If the investment policy states otherwise, this factor can be ignored since the money market will then be treated as an asset class just like bonds.

table 1

PORTFOLIO COMPOSITION AS OF DECEMBER 31, 1995

Issuer	Maturity	Coupon	Principal
Canada	1997/07/01	7.50%	$2,500,000
Canada	2000/03/01	8.50%	$3,000,000
Canada	2025/06/01	9.00%	$2,200,000
British Columbia	2005/08/05	8.00%	$1,200,000
Ontario	2025/12/02	8.50%	$2,000,000
Province of Quebec*	2000/09/09	10.50%	$1,000,000
Province of Quebec**	2000/10/16	10.00%	$800,000
Montreal***	2001/10/02	10.50%	$1,400,000
Toronto	2000/12/06	11.625%	$1,000,000

* Callable bond (callable date: 1996/09/09)
** Callable bond (callable date: 1996/10/16)
*** On April 1, the principal of this bond is increased to $2,500,000, following the investment of the amount accumulated in the cash position.

Yield curve movements

Factor 4 – The Curve Factor: Profits (losses) resulting from movements across the entire yield curve.

To estimate the impact of parallel shifts in the yield curve of the federal bond market, as well as changes in its curvature and twists on the portfolio's return, we estimate first the value that the bond portfolio would have at the beginning of period *t* if all bonds carried the same yield to maturity as those of equivalent securities in the federal bond market. Two values are then computed for the portfolio as of the beginning of period *t*, the first using yields to maturity in the federal bond market as of the beginning of the period, and the second using yields to maturity in that market as of the end of the period. The difference between these two values reflects the impact of movements in the federal bond market yield structure.

Changes in spreads

Factor 5 – The Spread Factor: Profits (losses) arising from variations in the spreads between short-, medium- and long-term bonds of the same categories as those included in the portfolio in relation to variations in the corresponding spreads in the federal bond market.[4]

To calculate these profits (losses), we begin by estimating the excess value[5] of the portfolio, at the beginning of period *t*, attributable to the difference between the observed federal bond market yield curve as of the beginning of period *t*+1 and the yield curve in the market for the categories of bonds included in the portfolio at that same point in time. We then estimate again the excess value of the portfolio also as of the beginning of period *t*, but this time using the same yield curves as before as of the beginning of period *t*. The difference between these two values is equivalent to the change in the portfolio's value caused by changes in the yield spreads during the period. Factors 4 and 5 together reveal, for "conventional" bonds, the impact of duration and convexity with regard to variations in the term structure. To extend our analysis to "exotic" bonds, like callable bonds, the following factor must be considered.

Security selection

Factor 6 – The Options Factor: Profits (losses) resulting from variations in the value of embedded derivative products in "exotic" bonds.

A good example of such embedded options is found in the callable and conversion provisions included in some bond issues. Analytically speaking, the combination

of these two features in a bond represents a type of embedded barrier option, where the payoff depends on whether the price of the underlying asset reaches a given barrier before a certain date.

The profits (losses) resulting from such embedded options are calculated by subtracting the additional portfolio value as of the end of period *t*, resulting from embedded options, from its value as of the beginning of the period. This difference gives the variation in portfolio value caused by changes in the value of embedded options during period *t*.

Factor 7 – The Exchange Factor: Profits (losses) from investments in bonds denominated in foreign currencies resulting from exchange rate variations of these currencies.

The profits (losses) attributable to this factor are estimated by measuring the impact (in Canadian dollars) of exchange rate variations on positions in bonds denominated in foreign currencies during period *t*.

Factor 8 – The Arbitrage Factor: Profits (losses) from arbitrage trading in the bond spot market.

Arbitrage opportunities arise in the bond spot market when the yield implicit in the price of a bond is higher than that implicit in the price of another bond of the same duration and rating. This situation can arise when a bond is temporarily "neglected" by the market. A simple arbitrage strategy would then consist in buying the undervalued bond and simultaneously selling (or shorting) the lower yield bond without modifying the duration or rating of the portfolio. This strategy will generate profits or losses used in the calculation of the return attributable to this factor.

Factor 9 – The Derivatives Factor: Profits (losses) from transactions in interest rate derivative products.

Interest rate derivatives are securities whose payoffs are a function of movements in specific interest rates. They include a variety of products, some of which are traded on exchanges (e.g. bond options and bond futures), while others are over the counter (e.g. swap options and interest rate caps). Bond portfolio managers often prefer to envisage the yield curve as being made up of segments of different maturities. The reason for this is that rate volatility across the various segments of short, medium and long maturities are not closely correlated; sometimes they may even move in opposite directions. Thus, if the portfolio manager takes a long position in a particular segment of the yield curve and finds later that this position is overexposed to the risk of parallel shifts in rates, he can use interest rate derivatives to reduce the exposure by reducing the duration of the investment. Profits (losses) resulting from transactions in derivative products during period *t*, capitalized to the end of the period, are used in the measurement of the return generated by this factor.[6]

The majority of bond portfolio managers do not resort to arbitrage or derivatives strategies as a means of enhancing the return of their portfolios. Factors 8 and 9 can be safely ignored when analyzing the performance attribution of these portfolios.

Factor 10 – The Selectivity Factor: Profits (losses) from net selectivity of securities.

The return attributable to these profits (losses) is estimated by dividing the fraction of return attributable to profits (losses) from net selectivity for period *t* by

table 2

MARKET VALUES AND CAPITALIZED PROFITS (LOSSES) OF THE PORTFOLIO FOR THE PERIOD JANUARY TO JUNE (C$)

	January	February	March	April	May	June
Total market value of bonds at month end	16,391,032	16,242,439	16,136,715	17,084,903	17,185,792	17,176,820
Total market value of T-bills at month end	738,854	769,843	995,552	179,641	207,366	503,870
Total portfolio market value at month end	17,129,886	17,012,281	17,132,267	17,264,544	17,393,158	17,680,689
Capitalized profits (losses) from arbitrage trading during the month	14,512	-36,442	27,541	68,411	-2,246	-14,741
Capitalized profits (losses) from transactions in derivative products during the month	7,221	16,435	14,474	-564	29,214	67,778
Contribution at the beginning of the month	0	0	0	250,000	0	0

the market value of the portfolio at the beginning of the period. Net selectivity includes the impact of bond picking and trading on portfolio return. This implies that an increase in the volume of transactions during a specific period may result in greater volatility.

Factor 11 – The Residual Factor: Non-factor sources of return.

The return unexplained by the above 10 factors shows up in this factor. The above list shows that some factors are directly related to spread and term structure risks during the period examined while others are not. Indeed, income earned by bonds and interest on cash are factors related to the passage of time and are independent of interest rate variations during period *t*. On the other hand, factors 4 through 10 directly or indirectly reflect the risks of yield curve movements and changes in spreads. Yield curve movements account for most of the variance in return in a well-diversified bond portfolio (see Kuberek, 1995). The impact of the duration of the portfolio is reflected primarily in yield curve movements and spread changes. In this sense, it can be said that the factors relating to all these shifts implicitly incorporate a measure of duration and convexity.

In many cases a comparative performance attribution is more relevant for portfolio analysis. Indeed, by comparing the portfolio performance attributions to those of a benchmark we can assess the differences in the components of total holding period return attributed to each factor, thereby identifying the portfolio's relative strengths and weaknesses. Needless to say, the choice of an appropriate benchmark is of paramount importance for these comparisons.[7]

Application of the Model

To illustrate the operation of our model and highlight its distinguishing characteristics, we apply it to a bond portfolio composed of select securities as of December 31, 1995 (see Table 1).

Our purpose is to analyze the performance attribution of this portfolio for a period of six months from January to June using comparative performance evaluation. At December 31, the value of the portfolio was $17,000,000, of which $16,739,615 was invested in Canadian bonds and $620,385 was placed in Government of Canada 30-day T-bills. For simplicity, the relevant data for the portfolio for the period under study are summarized in Table 2. Since all bonds in the portfolio are in Canadian dollars, the exchange rate factor is not included in the illustration.

We break down the holding period return for this portfolio using the factor model explained above. Table 3 sets out the results of these calculations.

For the purpose of this illustration, we use the Scotia McLeod Universe Index as the benchmark to measure the portfolio manager's added value and attribute it to the different factors explaining the holding period return. To this end, we begin by breaking down the holding period return of this benchmark for the six months under study as shown in Table 4. Note that there are no entries for the benchmark in the options, derivatives and arbitrage factors.

By combining Tables 3 and 4, we measure the value added by the manager and attribute it to the various factors as illustrated in Table 5.

Table 5 shows that overall, the portfolio under analysis outperformed the benchmark by 0.881%

table 3

PERFORMANCE ATTRIBUTION OF THE PORTFOLIO FOR THE PERIOD JANUARY TO JUNE

	January	February	March	April	May	June	Total
Return	0.764%	-0.687%	0.705%	-0.677%	0.745%	1.653%	2.495%
Time	-0.100%	-0.102%	-0.096%	-0.087%	-0.098%	-0.102%	-0.584%
Coupon	0.662%	0.605%	0.667%	0.632%	0.703%	0.680%	3.975%
Cash	0.018%	0.018%	0.022%	0.002%	0.004%	0.008%	0.071%
Curve	0.196%	-2.382%	-0.571%	-0.573%	0.561%	0.108%	-2.656%
Spread	0.797%	-0.531%	0.119%	-0.058%	0.013%	0.179%	0.514%
Options	-0.072%	0.172%	-0.013%	0.054%	-0.041%	-0.158%	-0.058%
Arbitrage	0.085%	-0.213%	0.162%	0.394%	-0.013%	-0.085%	0.330%
Derivative	0.043%	0.096%	0.085%	-0.003%	0.169%	0.390%	0.780%
Selectivity	-0.864%	1.652%	0.331%	-1.038%	-0.553%	0.633%	0.143%
Residual	0.006%	0.043%	0.002%	0.007%	0.002%	-0.009%	0.051%

table 4

PERFORMANCE ATTRIBUTION OF THE BENCHMARK FOR THE PERIOD JANUARY TO JUNE

	January	February	March	April	May	June	Total
Return	1.120%	-1.640%	0.050%	0.050%	1.010%	1.040%	1.613%
Time	-0.099%	-0.102%	-0.088%	-0.078%	-0.072%	-0.073%	-0.509%
Coupon	0.673%	0.623%	0.679%	0.663%	0.685%	0.664%	4.052%
Cash	0.000%	0.000%	0.000%	0.000%	0.000%	0.000%	0.000%
Curve	0.653%	-2.319%	-0.849%	-0.633%	0.362%	0.448%	-2.350%
Spread	-0.014%	0.093%	0.132%	-0.015%	0.017%	0.027%	0.239%
Options	0.000%	0.000%	0.000%	0.000%	0.000%	0.000%	0.000%
Arbitrage	0.000%	0.000%	0.000%	0.000%	0.000%	0.000%	0.000%
Derivative	0.000%	0.000%	0.000%	0.000%	0.000%	0.000%	0.000%
Selectivity	-0.093%	0.066%	0.175%	0.113%	0.018%	-0.022%	0.257%
Residual	-0.002%	0.015%	0.006%	0.004%	-0.003%	-0.003%	0.017%

over the period examined. This added value came largely from transactions in interest rate derivatives which, on net, enhanced total return by 0.780%, followed by the profits from arbitrage trading which contributed 0.330% and by the spread factor which accounted for 0.275%.

On the other hand, the positioning of the portfolio on the yield curve had a negative effect on the portfolio return relative to the benchmark, amounting to -0.316%. This negative impact is a direct result of the fact that the portfolio's average maturity is greater than that of the benchmark. Thus, by positioning the portfolio closer to the long-term maturity, the manager stood to lose on the return associated with a rise in the yield curve, but more than offset this loss with the gain resulting from transactions in interest rate

table 5

COMPARATIVE PERFORMANCE ATTRIBUTION ANALYSIS OF THE PORTFOLIO RELATIVE TO THE BENCHMARK FOR THE PERIOD JANUARY TO JUNE

	January	February	March	April	May	June	Total
Return	-0.352%	0.969%	0.655%	-0.727%	-0.262%	0.607%	0.881%
Time	-0.001%	-0.001%	-0.008%	-0.009%	-0.027%	-0.029%	-0.074%
Coupon	-0.012%	-0.017%	-0.012%	-0.030%	0.018%	0.017%	-0.037%
Cash	0.018%	0.018%	0.022%	0.002%	0.004%	0.008%	0.071%
Curve	-0.454%	-0.065%	0.280%	0.061%	0.199%	-0.336%	-0.316%
Spread	0.811%	-0.624%	-0.013%	-0.043%	-0.004%	0.152%	0.275%
Options	-0.072%	0.172%	-0.013%	0.054%	-0.041%	-0.158%	-0.058%
Arbitrage	0.085%	-0.213%	0.162%	0.394%	-0.013%	-0.085%	0.330%
Derivative	0.043%	0.096%	0.085%	-0.003%	0.169%	0.390%	0.780%
Selectivity	-0.772%	1.584%	0.155%	-1.150%	-0.571%	0.655%	0.124%
Residual	0.008%	0.028%	-0.004%	0.003%	0.005%	-0.006%	0.034%

derivatives and from arbitrage trading. Further, the added value from amortization, from coupons, from interest earned on the cash position and from the change in value of embedded options had very little impact on the portfolio manager performance. The smallness of non-factor return shown in Tables 3 and 4 is evidence that the model incorporates all the relevant factors that impact bond return.

Concluding remarks

The model presented here is quite useful when several managers specialize in managing different asset classes in a portfolio. To apply the model, reliable information on the characteristics of the various securities included in the portfolio to be analyzed and of those included in the corresponding benchmark must be systematically processed and analyzed.

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Endnotes

1 The federal bond market is usually the most liquid market for bonds in Canada and represents the lowest credit risk. It will be used throughout this paper as a reference point for the estimation of the various market parameters used in the analysis.

2 To measure this factor for portfolios where numerous transactions take place or when the accrued interest data are not available for each transaction, the impact of the coupons can be approximated:

$$C_{t2p} = \frac{\sum_{l=1}^{L} V_{tl} \times IR_{tl}}{V_{tp} + M_{tp}} - C_{t1p}$$

Where: V_{tl} = market value of bond l at the beginning of period t
IR_{tl} = yield of bonds similar to bond l at the beginning of period t
C_{t1p} = attributable to the portion of the discount or premium amortized.
The first term on the right hand side of the equation is an approximation of the return generated by both forms of interest income (amortization and coupons) from which we subtract the impact of amortization to isolate the return attributable to coupons.

3 The same approach is used even if cash is combined with a position in derivative products.

4 This corresponding spread in the market includes that of all categories of bonds included in the portfolio. In the example presented in section III, these categories include provincial, municipal and corporate bonds.

5 This additional value is positive if the reference market's yields are, on the whole, lower then those of the associated markets. Otherwise, it is negative.

6 It should be noted however that this type of duration based hedging though simple, does not provide a perfect hedge since it addresses only parallel shifts in the yield curve and does not take into consideration the impact of convexity.

7 Moreover, such a benchmark should particularly represent a feasible, unambiguous and measurable alternative to the portfolio under analysis.