Target PRACTICE

Using risk budgeting properly can help a portfolio zero in on—and meet—its risk and return objectives.

BY JOHN McNAIR

With the significant equity market downturn of the second half of 2008 and the subsequent upward rally of 2009, we are all spending more time analyzing the various risks embedded within our portfolios. We are all wondering whether or not there is a more effective way to manage and/or allocate risk. Perhaps risk budgeting can help.

Risk budgeting is a familiar *concept* among financial professionals but due to the numerous and obscure definitions for risk budgeting and due to the seemingly complicated calculations required to build such a framework, I would argue that it has not become a familiar *tool* among financial professionals. The first objective of this paper is to eliminate the obscurity by providing a rather simple definition for risk budgeting. The second objective is to provide you with some additive risks that can be easily calculated no matter how you wish to disaggregate your portfolio; and the third objective is to provide a few risk budgeting examples that can be immediately implemented.

DEFINITION 1.1*:

Risk budgeting is a framework that helps achieve a target portfolio ex-ante risk[1] by budgeting proportions of this risk to various portfolio components.[2]

Definition 1.1 suggests that you must first come up with a reasonable target ex-ante risk for your overall portfolio and then you must decide how this risk should be budgeted to the various components of your overall portfolio. For example, if you are a plan sponsor, you may wish to disaggregate your plan into asset classes and choose to budget 50% to 70% of the overall risk to the equity component of the fund, 30% to 50% to the fixed income component and 0% to 20% to the remaining asset classes. Once you have set your budgets, you must then calculate the risks of the asset classes based on their current allocations and then you must reallocate your portfolio until all risk budgets are satisfied. This approach can be quite tedious because unlike a typical budgeting exercise, such as budgeting for expenses, mainstream risks for components (asset classes in the above example), such as standard deviation and tracking error of returns, will not add up to their total portfolio counterparts. The reason for this is that these risks are not additive and, more often than not, the risk budgeting exercise becomes iterative and usually results in coming up with new risk budgets. This is not an ideal approach.

The next section will provide an overview of risks that are additive. To be more precise, Equations 1.1 to 1.2 (defined below) can be used to calculate the risk of any component of your portfolio and the sum of these component risks will add up to the total portfolio risk. This dramatically simplifies the risk budgeting effort and as you will see later in this paper, using these risks to actively rebalance your portfolio can improve your

John McNair is Senior Vice President and Head of Investment Research at Highstreet Asset Management.

portfolio's ability to meet its risk and return objectives. These component risks can also be studied individually to determine their relative riskiness and when aligned with their historical returns you will now know whether or not additional risk has been rewarded.

ADDITIVE RISKS:

Portfolio Variance (the square of portfolio standard deviation) and Portfolio Surplus Variance (the square of portfolio surplus volatility) can be disaggregated into additive Variance for portfolio components:

Equation 1.1:

$$\sigma_P^2 = \underset{\sim}{\omega}_P' \Sigma \underset{\sim}{\omega}_P = \begin{cases} \underset{\sim}{\omega}_P' \Sigma_1 \omega_{P_1}, \text{Variance for component 1} \\ + \\ \underset{\sim}{\omega}_P' \Sigma_2 \omega_{P_2}, \text{Variance for component 2} \\ +\ldots \\ \underset{\sim}{\omega}_P' \Sigma_n \omega_{P_n}, \text{Variance for component n} \end{cases}$$

Where:

$\underset{\sim}{\omega}_P'$, $\underset{\sim}{\omega}_P$: Row and column arrays of portfolio component weights
Σ : Covariance matrix of returns
Σ_i : ith column of covariance matrix of returns
ω_{P_i} : Portfolio weight for component i

Portfolio Excess Variance (Tracking Error squared) can also be disaggregated into additive Excess Variance for portfolio components:

Equation 1.2:

$$\sigma_A^2 = \sigma_P^2 + \sigma_B^2 - 2\underset{\sim}{\omega}_P' \Sigma_{P/B} \underset{\sim}{\omega}_B = \begin{cases} \sigma_{P_1}^2 + \sigma_{B_1}^2 - 2\underset{\sim}{\omega}_P' \Sigma_{(P/B)_1} \omega_{B_1}, \\ \text{Excess Variance for component 1} \\ + \\ \sigma_{P_2}^2 + \sigma_{B_2}^2 - 2\underset{\sim}{\omega}_P' \Sigma_{(P/B)_2} \omega_{B_2}, \\ \text{Excess Variance for component 2} \\ +\ldots \\ \sigma_{P_n}^2 + \sigma_{B_n}^2 - 2\underset{\sim}{\omega}_P' \Sigma_{(P/B)_n} \omega_{B_n}, \\ \text{Excess Variance for component n} \end{cases}$$

Where:

σ_P^2, σ_B^2 : Variance of portfolio and benchmark returns respectively
$\sigma_{P_i}^2, \sigma_{B_i}^2$: Variance of returns for portfolio and benchmark component i
$\underset{\sim}{\omega}_P'$: Row array of portfolio component weights
$\underset{\sim}{\omega}_B$: Column array of benchmark component weights
ω_{Bi} : Benchmark weight for component i
$\Sigma_{P/B}$: Covariance matrix of portfolio and benchmark component returns
$\Sigma_{(P/B)_i}$: ith column of above matrix

Let us now use Equation 1.1 in order to understand how much of the overall portfolio risk is attributed to each manager responsible for the Canadian equity portion of a balanced fund.

PORTFOLIO VARIANCE DISAGGREGATION EXAMPLE

Let's assume that your Canadian Equity portfolio is managed by two investment managers. Manager A is currently responsible for 45% of the total portfolio's allocation and Manager B is responsible for 55%. Let's consider the two-year ex-ante period ending June 30, 2009. To calculate component Variances you must first calculate Manager A's and B's Variance of monthly returns and their covariance of monthly returns:

- Manager A's Variance of monthly returns = 0.22%
- Manager B's Variance of monthly returns = 0.10%
- Covariance of monthly returns between Manager A and B = 0.11%

Using Equation 1.1, we now know that:

$$\Sigma = \begin{bmatrix} \sigma_A^2 & \sigma_{AB} \\ \sigma_{AB} & \sigma_B^2 \end{bmatrix} = \begin{bmatrix} 0.22\% & 0.11\% \\ 0.11\% & 0.10\% \end{bmatrix}$$

Therefore:

- Manager A's component Variance = $\underset{\sim}{\omega}_P' \Sigma_1 \omega_{P_1}$ = [45%(0.22%)+55%(0.11%)]45% = 0.07%
- Manager B's component Variance = $\underset{\sim}{\omega}_P' \Sigma_2 \omega_{P_2}$ = [45%(0.11%)+55%(0.10%)]55%=0.06%

- Total Canadian Equity portfolio Variance = 0.07%+0.06% = 0.13%
- Manager A's portion of Canadian Equity portfolio Variance is 0.07%/0.13% = 56%
- Manager B's portion of Canadian Equity portfolio Variance is 0.06%/0.13% = 44%

This type of analysis can be quite revealing. Even with 10% less allocation than Manager B, Manager A's proportion of the overall Canadian Equity portfolio Variance is more than 10% greater than that of Manager B. If this was not what you expected when you set the allocation to each Manager, then you may need to introduce a risk budgeting framework in order to budget an amount of risk for each. For instance, a 40% allocation to Manager A and a 60% allocation to Manager B would equalize their component Variances.

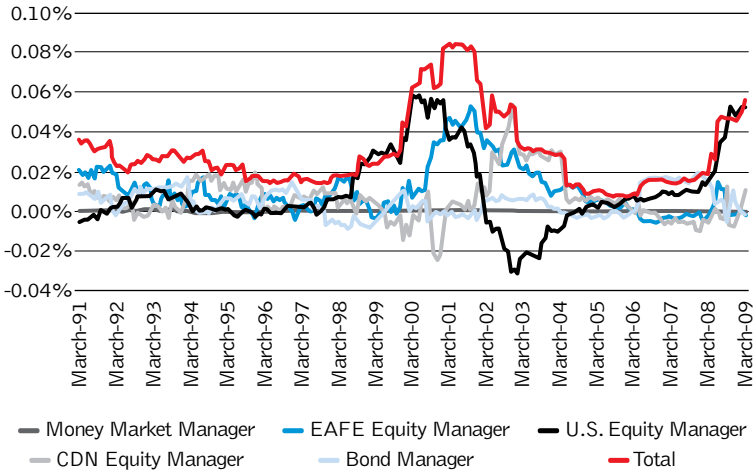
The above analysis raises awareness on the component risks at one point in time. To gain more insight from component risk analysis it is important to examine how they have changed over time. Let us now consider how the 2-year rolling version of Equation 1.2 has changed over time for a pension plan with one Manager[3] per asset class and whose allocations to these asset classes are kept constant and equal to that of its benchmark:

- 5% DEX 91 Day T-Bill Index
- 35% DEX Universe Bond Index
- 30% S&P/TSX Composite Total Return Index (TSX)
- 15% S&P 500 Total Return Index (S&P500)
- 15% MSCI EAFE Total Return Index (EAFE)

Figure 1 exhibits the 2-year rolling ex-post component Excess Variances for the various Managers responsible for the active management of the pension plan from April 30, 1989 to March 31, 2009. Figure 1 demonstrates that with constant portfolio allocations, the component Excess Variances for the portfolio are definitely not constant. The Canadian Equity portfolio weight is twice that of the US Equity and the EAFE Equity portfolio weight, but during the tech collapse its component Excess Variance of returns was significantly less than that of its foreign counterparts. Was this due to a style drift? Did the manager deploy cash?

These types of charts will raise awareness as to what has been the least/most risky component of the portfolio throughout the investment horizon and whether or not these risks have been consistently rewarded in the form of additional return to the portfolio. Armed with these charts, you will also know which components of the portfolio need to be monitored more closely in order to achieve plan objectives on an ex-post basis.

FIGURE 1: 2-Year Rolling Component Excess Variance



For the next section let's assume that you are the plan sponsor responsible for the oversight of the pension plan defined above and on a monthly basis the performance and risk of the pension plan must meet the following objectives.

PENSION PLAN EX-POST OBJECTIVES

Performance Objective: 2-year rolling annualized return must be 1% greater than that of the benchmark.

Risk Objective: 2-year rolling Excess Variance (Equation 1.2) must be kept in the range of 0.05% to 0.15% (equivalent to 2% to 4% annualized tracking error).

Asset class allocation constraints:

- Equity allocation must be kept in the range of 40% to 70%
- Cash allocation must be less than 10%
- No short selling or leverage

To meet these objectives you have decided to make use of the following risk budgeting framework:

RISK BUDGETING FRAMEWORK

Every quarter the pension plan's manager allocations will be monitored and potentially changed so that the following 2-year rolling ex-ante risk budgets are satisfied:

- Component Excess Variance vs. the benchmark from equity managers must represent more than 60% of the total pension plan's Excess Variance vs. the benchmark.
- Total pension plan Excess Variance vs. the benchmark to be kept in the range of 0.05% to 0.15%.

Let us examine how the above framework helped achieve the pension plan's objectives by comparing this framework to a constant mix framework that ensures that the

allocations to the various managers are kept constant and equal to that of the benchmark.

Figure 2a exhibits the 2-year rolling ex-post total pension plan Excess Variance for the constant mix framework and for the risk budgeted framework and Figure 2b exhibits the 2-year rolling ex-post total pension plan Excess Return for the constant mix framework and for the risk budgeted framework. The constant mix approach does a poor job at meeting both risk and return objectives whereas with risk budgeting, both the risk and return objectives are met with a higher frequency:

Constant mix results:

- Risk Objective is met in only 15% of the rolling 2-year observations
- Performance Objective is met in only 31% of the rolling 2-year observations

Risk budgeting results:

- Risk Objective is met in 65% of the rolling 2-year observations
- Performance Objective is met in 46% of the rolling 2-year observations

As can be seen from these results, the risk budgeting approach improves the frequency with which both objectives are met. Furthermore, the risk budgeting approach outperforms the constant mix approach by almost 1% annualized with a superior information ratio[4] (0.42 vs. 0.33) from April 30, 1989 to March 31, 2009 with only 11% more turnover than the constant mix approach.

Now let's consider an example where we remove all of the asset class allocation constraints from the pension plan objectives and allow risk budgeting to completely dictate how the portfolio assets will be allocated. In this example, one asset class such as equities or even one manager could represent 100% of the portfolio.

Risk budgeting results:

- Risk Objective is met in 82% of the rolling 2-year observations
- Performance Objective is met in 30% of the rolling 2-year observations

When the asset class allocation constraints are lifted, the risk budgeting approach can only improve upon the frequency of maintaining the Excess Variance in the range of 0.05% to 0.15%. Furthermore, the risk budgeting approach underperforms the constant mix approach by almost 1% annualized with an inferior information ratio (-0.13 vs. 0.33) over the entire test period.

FIGURE 2a: 2-Year Rolling Excess Variance

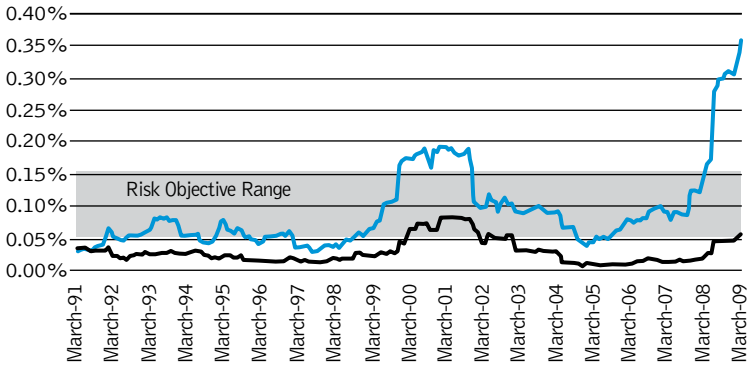
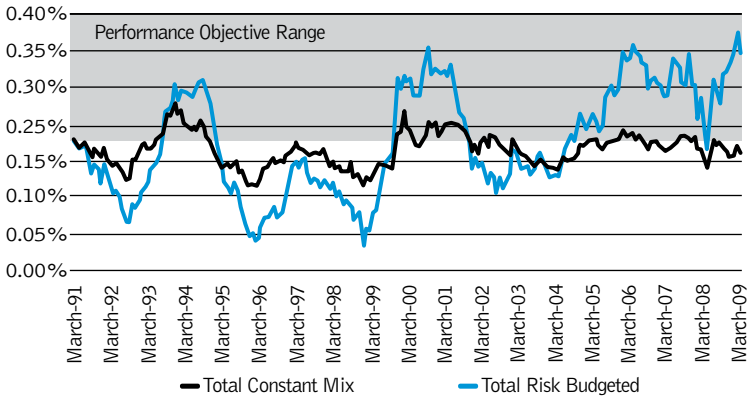


FIGURE 2b: 2-Year Rolling Excess Return



CONCLUSIONS

Risk budgeting is a framework that enables risk allocation and risk assessment. Examining component risks over time will raise awareness as to what has been the least/most risky subset of the portfolio and whether or not these risk exposures have been adequately compensated with additional return to the portfolio. Allowing for additive risks, the risk budgeting framework is straightforward. It is, however, a paradigm shift compared to the traditional effort of allocating proportions of total assets instead of amounts of risk to the various components of a portfolio. From the above examples we see that making use of risk budgeting with a well-disciplined rebalancing process shows promise in improving the portfolio's ability to meet its risk and return objectives. ▮

FOOTNOTES

1. A risk that is calculated using results obtained in the past.
2. Portfolio subsets such as asset classes, GICS sectors etc.

* It is important to note that risk budgeting does not guarantee realized (ex-post) risk.

3. Median managers were selected from Mercer's Manager Performance Analytics database.
4. Excess return vs. the benchmark divided by the standard deviation of relative returns vs. the benchmark.