# WHICH HEDGE FUND STRATEGIES?

*New tool for portfolio allocation across hedge fund strategies.*

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How much capital should investors allocate to different hedge fund strategies? The answer is elusive. Hedge funds exhibit complex, non-normal return distributions. In this context, it is difficult to use standard mean-variance portfolio theory and performance measures based on it (e.g. the Sharpe ratio). An allocation technique based on Polynomial Goal Programming (PGP) appears to be a suitable alternative. It can be used by fund of hedge funds managers and by large institutional investors such as pension funds that are investing in both hedge funds and traditional asset classes.

To start, the investor constructs representative portfolios of each hedge fund strategy. These should be consistent with the total investment size and should reflect the set of investable hedge funds (i.e. non-closed funds that satisfy certain suitability criteria). For instance, large investors might construct portfolios of 15 funds per strategy, whereas small investors might construct portfolios of only five funds per strategy. This creates a "like for like" basis for investment decisions. It should be noted that widely reported strategy indexes often do not share this property—each index has a different number of component funds. A strategy index based on fewer funds will have higher variance, all else equal, and will be (incorrectly) assigned less capital.

Next, the investor specifies the return distribution of each representative portfolio, based on historical data modified to reflect future expectations. In addition to these representative portfolios, the analysis assumes the investor can borrow and lend at a risk-free rate. The asset space can also be extended to include stock and bond portfolios.

Within this asset space, the PGP method selects the asset weights which balance the competing objectives: maximizing expected return while simultaneously minimizing variance, maximizing skewness and minimizing kurtosis. This is a two-step process.

In the first step, the investor solves for the maximum value of expected return ($Z_1$) obtainable holding variance constant at one, and disregarding his preferences for the other return moments. Analogously, by holding variance constant at one, the investor also solves for the maximum 3rd return moment ($Z_3$) and the minimum 4th return moment ($Z_4$). Note that holding variance constant at this stage is harmless since the final stage risky portfolio weights can be rescaled to obtain the desired variance.

In the second step, the investor solves for the allocation weights that minimize:

$$Z = (1+d_1)^a + (1+d_3)^b + (1+d_4)^c$$

where $a$, $b$, and $c$ represent the investor's preferences for expected return, the 3rd return moment (or skewness), and the 4th return moment (or kurtosis); $d_1$ is the difference between the expected return of the investor's portfolio given his allocation weights and the "optimal" expected return $Z_1$ found in the first stage; analogously, $d_3$ and $d_4$ are the differences between the 3rd and 4th moments of the investor's portfolio, and the values of $Z_3$ and $Z_4$, respectively.

The beauty of this method is that it requires the investor to specify only three preference parameters, rather than an entire utility function. It also can be easily extended to incorporate many real-life constraints, such as minimum investment restrictions.

As a final note, the return distribution of a strategy in isolation does not provide a complete picture. Just as co-variance matters most in standard mean-variance frameworks, co-skewness and co-kurtosis between different assets matter here. Davies et al. (2005) find that equity market neutral and global macro funds have especially important roles because of their interaction with other assets. ■

## References:

Davies, R.J., Kat, H.M., Lu, S. (2005). Fund of hedge funds portfolio selection: A multiple-objective approach. Available at: http://ssrn.com/abstract=476862.