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MARKET RISK

How investor beliefs impact market volatility.

BY HORACE "WOODY" BROCK

A hallmark of progress in 20th century science was a series of advances in our understanding of the nature and role of uncertainty in physics, information theory, game theory, and economics. In economics—finance in particular—new research at Stanford University in the latter 1990s has made possible a complete reinterpretation of the phenomenon of "market volatility" in finance. It is the purpose of this short note to summarize the essence of this advance made possible by the new Theory of Rational Beliefs developed by the Stanford mathematical economist Mordecai Kurz. In Section 1, I shall discuss the decomposition of overall market risk into two components: exogenous and endogenous risk. In Section 2, I shall briefly extend the analysis in a way that helps us understand another dimension of market risk that is rarely recognized as risk, namely the uncertainty posed by the existence of long-cycles (e.g., alternating bull and bear markets).

1) TWO TYPES OF RISK

Traditionally, market risk has been external or "exogenous" in nature. Specifically, prices of securities move in sync with and in proportion to "news" about fundamentals. In other words, market volatility is exogenously driven. This is the case with all standard pricing models and in the Efficient Market Theory in general. One of the earliest realizations that there is more to risk than the impact of exogenous events came with the path-breaking work of Robert Shiller in the early 1980s. Shiller showed that, over a very long sample period, exogenous events ("news") could only explain about 25% of observed market risk. The great question then became: "Where does the other 75% of risk come from?" This and related questions (e.g., why is the observed equity risk premium over five times as large as that predicted by the Efficient Market Theory?) led to two decades of work in which various efforts were made to extend economic theory so as to account for real-world levels of observed volatility. We thus witnessed the advent of "noise trading" theory, behavioural finance, and other approaches to explaining volatility. Most of these efforts assumed at one level or another that the principal source of excess volatility was investor irrationality.

The new Theory of Rational Beliefs approaches matters in a very different manner. It retains the assumption that people are rational in the restricted sense that they attempt to make decisions (and construct portfolios) guided by the principle of maximizing expected risk-adjusted returns. However, unlike in classical theory, the new theory posits a non-stationary environment in which agents cannot know the correct probabilities of future events and are thus wrong when forecasting the future. Not irrational, but wrong. Moreover, the mistakes of agents are often correlated, and when mistakes are correlated, it is easy to show that excess market volatility results. (For everyone will shift their portfolios in the same direction.) In other words, a principal source of market volatility lies in the reaction of investors to the mistakes that they end up making because their prior beliefs about the news were wrong.

To sum up, it is the belief structure of the market—in particular the intersection of the prior belief structure

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about the news with the actual news—that is what really matters. Market volatility resulting from this phenomenon is now known as endogenous uncertainty. Total volatility can be shown to be the sum of exogenous volatility (the volatility that would result were there no mistakes) and endogenous volatility. Kurz has shown that the magnitude of the latter will be at least three times that of the former, thus laying the Shiller paradox to rest. Moreover, Kurz has been able to formalize all this within the bedrock of general equilibrium theory. As a result, these intuitively appealing ideas now rest on very solid conceptual and mathematical foundations.

There is a bit more to this story than we have suggested just above, and it will be helpful to formalize the distinction between classical risk that arises in Efficient Markets Theory with the new generalized concept of total risk. To do so we need to introduce some simple notation. The classical theory posits that the volatility of the market is given by a known function F of the random variable x (news) plus an error term ε, i.e.

$$\{p\} = F(\{x\}) + \varepsilon$$

where the curly brackets refer to a known probability distribution of x and hence (through an elementary transformation of variables) a known distribution of price p. The standard deviation of {p} will correspond to the usual meaning of market risk. The assumptions here are very strong indeed, and have been formalized in the concept of a Rational Expectations Equilibrium: All agents are assumed to know and agree upon the "true" probability distribution {x} of all future news and upon the nature of the function F that maps such news into price. Those early scholars who gave us this theory were never clear on how such conditions could ever arise in reality, but Kurz has made this clear: The stochastic process generating both the news and the price given the news must be a stationary stochastic process, i.e. a process that is non-time-varying. If this is true, then by law-of-large-numbers inductive reasoning, all agents would in principle be able to learn the true nature of this stochastic process. Importantly, in such a world, the concept of "mistake" cannot even be defined. By extension, mistakes cannot matter to prices. In the new theory, mistakes are everything.

The Rational Beliefs model generalizes (1) in many different ways. In particular, it assumes that the economic and financial environment is partly non-stationary, or in more commonplace terms, that "structural changes" periodically occur (e.g., the advent of OPEC in 1973) that cause the dynamical behaviour of economies and markets over time "to rhyme, not repeat" as historians often say. The precise way in which and timing with which these structural changes unfold cannot be known from the data given non-stationarity, and as a result virtually all investors' forecasts are wrong. And as a further result, most asset prices are wrong compared to what they ideally would be under the stationary model (1).

The new theory brings us much closer to reality than the classical theory, and interestingly it includes at a fundamental level the concept of investors' second-guessing each other that Keynes first introduced in his celebrated Beauty Contest paradigm. Specifically, we have the following generalization of (1):

$$\{p\}^* = F^*(\{B(M)\}, \{B(x^c)\}, x) + \varepsilon$$

where the true probability of price $\{p\}^*$ is a function of the three entities within the rounded parentheses. (For reasons that will soon become clear, no one investor can possibly know this true probability, so all forecasts of it will be wrong.) The first term represents the distribution of beliefs across investors as to the true nature of the pricing model M that maps news into price. No one agrees on the nature of this map, but each investor utilizes some such map in arriving at his/her forecast of future price given his/her subjective probability distribution over future news. In short, the first term {B(M)} represents the role of Pricing Model Uncertainty in impacting future market prices through the function F*. It can be shown that the greater this type of uncertainty is, the greater market overshoot will be. The case of currencies, growth stocks, and emerging markets comes to mind: Investors agree that virtually no one knows how to correctly "price" such news in these markets. They thus should be very difficult to forecast and quite volatile as well. Assumptions of investor irrationality play no role in these assertions at all.

The second term $\{B(x^c)\}$ represents the distribution of beliefs across investors as to the likelihood of future news itself. This takes into account the fact that, in a

non-stationary world, investors legitimately disagree in their probabilistic forecasts of the future. Since there will only be one "truth" ex post, this is a way of stating that everyone will in general be wrong, and that all assets will thus be mispriced in general. However, there is one very subtle difference here when compared to model (1) above. The superscript e next to x denotes that the news that matters includes not only what will happen to "fundamental variables" such as earnings and inflation, but also the expectations of each investor as to other investors' forecasts of such fundamentals. To be more technical, x in model (1) is the classical "state of the world tomorrow" variable introduced by Kenneth Arrow in his legendary 1953 paper on state preference theory. In this model, tomorrow's prices depend only on the external state of the world tomorrow. In contrast, xe in (2) is an extended state variable in which the "state" that matters consists not only of what will happen tomorrow when the exogenous news is announced but also of what others today expect that news to be. This is where the Beauty Contest idea of Keynes enters in.

Finally, note the appearance of x itself as the third term within the parentheses. This represents the actual ex post realization of the news tomorrow when it is announced. What the map F* in (2) is thus telling us is that the market price that results tomorrow will depend jointly upon (i) the distributions of investor expectations about future price as determined by the first two terms, and by (ii) what actually happens in the news. It is the interaction of these two dimensions of the problems that determine how many people end up how wrong in what direction, and this in turn causes a "reaction" both in quantity of shares traded and in equilibrium price.

It goes without saying that the ability to mathematize all this and to incorporate it within a bona fide general equilibrium model retaining the assumption that investors are rational in attempting to do their best is one of the great triumphs in the history of economic theory. Regrettably, the mathematics involved is formidable, and this is one reason the theory is currently little known. For those interested in a comparable feat within the social sciences, there is a very good analogy between Kurz's work and what happened within game theory forty years ago. By the mid-1960s, classical game theory was proving disappointing since it made predictions that were wrong. Once again, many assumed that this proved that people are irrational. But John Harsanyi at the University of California at Berkeley arrived at a radically different conclusion: He retained the assumption that agents are rational, but argued that classical game theory required players to know far more than they ever could about the structure of the game. In particular, he asked: How would agent i possibly know the risk attitude of player j, and thus how could each know the "payoff matrix" of the game perfectly as had always been assumed? (Recall that the payoff matrix in game theory consists of utility payoffs.)

Harsanyi thus developed his now celebrated theory of games with incomplete information, which incorporated ignorance by players about the "types" of their antagonists. The resulting theory worked well in practice, and earned him the Nobel Prize that he shared in 1994 with John Nash, Jr. and Reinhard Selten. Both Kurz and Harsanyi achieved the same result: They showed that the culprit in classical theory lay in unrealistic assumptions about what rational people could know, and not in putative irrationality. As a result, both classical theories ended up suppressing a principal source of real-world risk and uncertainty.

In applications, the Theory of Rational Beliefs can be shown to generate from first principles numerically accurate forecasts of the mean returns and variances of bills, bond, and stocks, and the empirical value of the equity risk premium. It can also explain such phenomena of GARCH, e.g., time-varying variances of precisely the kind we observed in global markets during May-June 2006, and can also explain why forward rates in the currency markets are systematically biased as predictors of future spot rates. What matters in all cases is the prior distribution of forecasts in relation to the truth about the news that eventuates. It is the resulting distribution of mistakes that in turn determines the magnitude of the market reaction to news. For example, it should now be clear to the reader how the same news on two different occasions can generate completely different price changes due to differences in prior expectations about such news—something that could never occur in classical theory.

THE NEW THEORY BRINGS US MUCH CLOSER TO REALITY THAN THE CLASSICAL THEORY.

2) Long-Cycle Theory

The present author has applied the new theory described above to explaining the existence and import of long bull/bear market valuation cycles—another real-world phenomenon that is verboten in classical theory. There are two questions that arise in this context. First, what causes cycles in valuations whereby P/E ratios in stock markets trend from 8 to 24 and back over time? Second, what does this phenomenon have to do with "risk"? Let us start with this second question. Virtually none of the many papers attempting to explain the high risk premium of equities incorporates the existence of long-cycles as part of risk. Yet in fact such cycles are arguably the single most important of all components of risk at a very fundamental level of analysis. To see why, recall what Modigliani and others taught us about savings behaviour. Why do we save? The answer is that we save to garner enough money by age 65 (or whenever we retire) to be able to continue enjoying the consumption stream we have become used to. But what is the principal risk we face in achieving this single goal over the forty-some years during which we save? It is that we cannot know when bull and bear market long-cycles will occur. Note that this constitutes a huge risk, having nothing to do with "volatility." Thus, a person who retired in 1966 and died fifteen years later in 1981 (exactly the case of the author's father) lost nearly 60% of his/her real wealth in stocks and bonds in the long bear market of that era.

The same person's younger brother who retired in 1981 and died in 2000 experienced real wealth gains of over 500% in a comparable Markowitz portfolio of stocks and bonds! Non-stationarity of the environment is the precise reason why no one can predict the timing of such trend-reversals. In a stationary environment, conversely, everyone would know the timing involved and much of the cycle's impact would thus be arbitraged away. The phenomenon we have described is a hidden dimension of risk that is one further reason why investors deserve and indeed get a large equity risk premium over the long run. But it is virtually never cited.

As for the first question—why long-cycles exist in the first place—Kurz and his colleagues have introduced the concept of "persistent" Belief Structures, and this concept goes a long way in explaining long-cycles. When equity markets start to proffer above-average returns to investors over a growing period of time (e.g., the returns of 1981-2000), investors start to fall in love with stocks and do so at an increasing rate. As more and more investors convert to return-optimism, their enthusiasm pushes the P/E ratio up ever further. Such optimism is self-reinforcing since rising P/E ratios make everyone involved even richer, even if the growth rate of earnings is in fact falling, as it did in 1997-2000. As a result still more investors shift into stocks. And vice versa in bear markets. In short, fluctuations in "persistent" optimistic-and-pessimistic belief structure regimes can go a long way in explaining long-cycles. Note that in an efficient market context no such phenomena would be observed.

THE CULPRIT IN CLASSICAL THEORY LAY IN UNREALISTIC ASSUMPTIONS ABOUT WHAT RATIONAL PEOPLE COULD KNOW.

This is because of the assumption that everyone knows the "true" stochastic process of returns perfectly. As a result, the elementary concept of being "optimistic" or "pessimistic" cannot meaningfully be defined and can thus play no role in such theories.

Hopefully, we have demonstrated the importance of the new Theory of Rational Beliefs with its emphasis on the concepts of investor Belief Structures, and the correlative concept of the distribution of investors' mistakes. As one final thought, once the role of mistakes is fully and formally acknowledged, the deleterious role of leverage in society becomes much more clear. Indeed, as this author has recently argued elsewhere, leverage in a world of ineluctable mistakes generates "externalities" (e.g., risks of a market meltdown) that cannot be properly "priced" by the market. Government intervention in regulating leverage is thus called for on the grounds of elementary welfare economics. However, due to the tyranny of "markets always know best" Efficient Market Theory dogma, this elementary point is not currently acknowledged either by most central banks or other arms of government. We fear that it soon will be! ▮

Endnotes

1. For a good summary of all this, please refer to Endogenous Economic Fluctuations: Studies in the Theory of Rational Beliefs, M. Kurz (ed.), Springer Series in Economic Theory, No. 6, Springer-Verlag, August 1997, and "Determinants of Stock Market Volatility and Risk Premia," M. Kurz, H. Jin, and M. Motolese, appearing in *Annals of Finance*, Vol. 1, 109-147, Springer-Verlag 2005.