Margin loans to Canadian investors totalled approximately $8 billion in the first half of 2002, and exceeded $10 billion at the start of 2001. This enthusiasm for leveraged investing is due to potential magnification of bull market profits. However, there is considerable risk from greater losses when stock prices fall. The technology sector has provided painful lessons to investors in recent years.

One prominent example is Nortel, which quadrupled in value from fall 1999 to its record highs above $120 in summer 2000. Investors buying on 50% margin approximately doubled their profits before interest charges. But, in the two years following its peak, Nortel plummeted to less than $2 per share by summer 2002. Buying Nortel on 50% margin in 2001 produced losses of almost 200% for investors who held the stock throughout the decline.

While this example highlights the risk inherent in buying a single stock on margin, it does not shed any light on how leverage affects diversified portfolios in the long run. If the stock market continues to have a long-term upward trend, as in the 20th century, leverage may increase investors’ profits.

Our research examines the risks and returns on diversified Canadian equity portfolios when margin is used throughout long periods, such as 10 or 20 years. How much leverage is prudent to use? What are the chances of underperforming a more conservative strategy? Should the asset allocations vary across different long-term horizons? This study provides answers to these questions.

Our data include monthly stock returns on the Canadian Financial Markets Research Centre value-weighted index, and Government of Canada 91-day Treasury bill returns. Each series contains 623 values over the period February 1950 through December 2001. The mean annualized stock return was 12.42%
while the Treasury bill mean was 6.15%. This seems to suggest huge profits would have been obtained from a highly leveraged portfolio over the 52 years, even if the margin loan rate was somewhat higher than the Treasury bill rate. But Figure 1 shows the opposite!

Figure 1 considers asset allocations ranging from 0% stock up to 400% stock. These stock proportions are shown in decimal form along the horizontal axis. For stock proportions below 1.0, the remainder of the allocation is invested in Treasury bills. Stock proportions above 1.0 are achieved by borrowing at a rate 2% higher than Treasury bills. Note that the current 30% margin requirement for large stocks limits the allocation to 333%, and a 50% margin reduces the maximum allocation to 200%.

To compute ending wealth, $1 is initially invested in 1950 at a specified stock proportion, with subsequent asset allocations held constant by rebalancing monthly. Stock is purchased or sold at the end of each month to restore the allocation to the initial proportion. For example, suppose a 150% stock portfolio grows to a $10 net value. This portfolio would consist of $15 in stock and a $5 margin loan. If the stock value grows to $20 the next month, the stock proportion declines to 133%, ignoring interest on the margin loan. The original 150% proportion is restored by purchasing another $2.50 in stock, increasing the margin debit to $7.50.

During a bull market, rebalancing is necessary to prevent a decline in leverage. After many years without rebalancing, the margin loan becomes so small relative to the stock value that the portfolio approaches a 100% stock allocation. The rebalancing is also useful and realistic for market downturns because it works like a margin call: a drop in stock value produces a higher leverage proportion. To restore the asset allocation after a market decline, some stock must be sold with the proceeds used to reduce the margin debit.

Along the vertical axis of Figure 1 we see that $1 invested in Treasury bills in 1950 grew to $22.13 by the end of 2001, adjusted for inflation. The 100% stock allocation produced $234.14, while 150% stock resulted in $278.55. But when further leverage increases the stock allocation above 150%, ending wealth drops, approaching zero at 400% stock.

While these wealth declines are puzzling at first glance, they reflect the powerful consequence of leverage during market declines. In October 1987, stocks declined by almost 25%. An investor who had maintained a 400% stock allocation since 1950 would be nearly bankrupt. When a portfolio is reduced to a tiny dollar value, even huge subsequent percentage gains will not restore the original dollar value.

Because leverage magnifies any losses, long-run returns decline as leverage increases. This is demonstrated by Ferguson (1994) using some simple binomial examples, and expanded and illustrated by Domian and Racine (2002) for U.S. stock portfolios. Domian and Racine show that the wealth declines are consistent with the maximum expected log (MEL) rule.

What inferences can be drawn about leveraged investing over horizons shorter than the full 52 years? We next discuss a methodology that allows us to investigate various strategies over investors' life cycles.

**Methodology**

We analyze some realistic leveraged investment strategies over various holding periods. These strategies involve either monthly contributions to an investment portfolio, or monthly withdrawals. The time horizons extend up to 40 years.

It is difficult to empirically assess long holding period results—regardless of leverage—because there are so few observations. For example, ending wealth could be computed from some strategy over January 1971 through December 1995, and again for January 1972 through December 1996. But these would not be independent observations because of the substantial overlap between the two periods. Even with 100 years of data, there are just four independent 25-year periods.

To resolve this difficulty, probability distributions can be obtained with the resampling approach presented by Butler and Domian (1991). The resampling technique typically uses real returns, since they are more stable over time. We use inflation data from CANSIM to compute real returns. To illustrate the resampling approach, consider a strategy of investing one real dollar every month throughout a designated investment period. The procedure is implemented as follows:

1. Randomly select one of the 623 months. Record the
observed real stock and T-bill returns for this month.

2. Compute the portfolio returns for asset allocations ranging from 0% stock up to 350% stock. Apply the return to the dollar value of the portfolio, which is $1 at the beginning of the first month.

3. Repeat the previous steps n x 12 times with replacement, each month contributing an additional dollar to the portfolio. Monthly rebalancing restores the asset allocation to the specified amount. Compound these monthly figures to construct ending wealth for one representative n-year (n = 1, 5, 10, 20, 30, 40) investment period return for each asset allocation.

4. Perform the entire procedure 500,000 times to generate n-year wealth distributions from the observed history of real monthly returns.

**Results**

Figure 2 displays the median ending wealth per dollar of monthly investment, over periods ranging from 10 to 40 years. The one-year and five-year periods are omitted for clarity since they plot close to the horizontal axis. Because ending wealth is calculated from real returns, the amounts are in real dollars. We use 36 asset allocations in 10-percentage-point increments: 0% stock, 10% stock, 20% stock, and so on, up to 350% stock.

The 10-year graph in the lowest portion of Figure 2 is fairly flat. A $1 monthly investment in Treasury bills (i.e., the 0.0 stock proportion) produces median ending wealth of $133. This rises to $172 with 100% stock, $180 with 200% stock, then declines to $167 with 300% stock.

The graphs become more arch-shaped as the investment period is lengthened. Median wealth over 20 years reaches a maximum of $555 with a 1.8 stock proportion, then declines by 34.2% to $365 for 350% stock. Maximum median wealth is $2,858 for a 40-year horizon, also at a 1.8 stock proportion. The 40-year wealth is reduced by 64.4% to $1,017 at 350% stock. Thus, if leverage is increased beyond the wealth-maximizing levels, the pronounced declines in ending wealth are amplified for longer investment periods.

Our results are expressed in terms of medians instead of means. Because of positive skewness, the means are substantially higher than the medians. For the 40-year holding period and 200% stock proportion, the median ending wealth in Figure 2 is $2,789, while the mean is $10,202. This mean is approximately the 80th percentile of the distribution. Due to the skewness of the distributions, medians are the more informative measure of ending wealth.

The resampling program can easily be modified to reflect other investment contribution patterns. A simple alternative is a constant growth rate in the dollar amount contributed.

Our findings do not imply that every investor should use leverage. Instead, they provide an upper boundary for the most aggressive investors. Investors who are more risk averse may decide to forego any leverage. Conservative investors may be particularly concerned about the lower percentiles of the probability distributions. One benchmark may be the chance that a leveraged stock portfolio will underperform Treasury bills (the 0% stock portfolio).

Further insights on downside risk come from the lowest percentiles of the wealth distributions. Consider a 1.8 stock proportion with constant $1 real monthly investments over 20 years (240 months) so the total investment is $240. The median ending wealth was shown in Figure 2 as $555. In contrast, the fifth percentile of the distribution is just $151, and the first percentile is only $93. These values imply that there is a 5% chance of losing at least $89 of the $240 investment, and a 1% chance of losing at least $147.

We end this section with a look at cash withdrawals instead of contributions. This is relevant later in the life cycle, when investors draw down their portfolios during retirement. The resampling procedure can easily be adapted for a variety of cash flow patterns. We consider
constant real monthly withdrawals, expressed as a percentage of the initial portfolio value. It is convenient to give these percentages in annual terms. For example, a 5% annual withdrawal rate from a $500,000 portfolio is $25,000 per year or $2,083 per month.

Even a modest withdrawal amount may not be sustainable over a long time horizon—the portfolio may simply run out of money. The probability of bankruptcy depends on the asset allocation, as well as the time period and withdrawal rate. Figure 3 shows bankruptcy probabilities for withdrawal periods ranging from five to 30 years. The solid lines show a 5% withdrawal rate, and dashed lines indicate a 10% rate.

There is little chance of running out of money over the five-year period. With a 5% withdrawal rate, the bankruptcy probability is less than 1% for stock proportions below 2.5. Obviously bankruptcy is more likely at the higher 10% withdrawal rate, but even here the chances are still small: less than 1% for up to 150% stock, and just 3.62% for the 200% stock allocation.

Conservative investors with modest withdrawal rates need not fear bankruptcy for either five or 10 years. A Treasury bill portfolio can provide a 10% withdrawal rate for 10 years, with only a 0.08% chance of bankruptcy. The five-year and 10-year results support the traditional view favouring conservative strategies in retirement.

There is a dramatic change in the graph for 20 years: now there is a 100% probability of bankruptcy for the lowest stock proportions at the 10% withdrawal rate. To avoid certain ruin, an investor wanting to maintain the high withdrawal rate would have to invest in equities, and may even consider leverage. The bankruptcy probability is 64.91% at a 1.0 stock proportion, and declines slightly to 59.86% at the 1.6 stock proportion.

In the 30-year period, there is nearly a 100% probability of bankruptcy from a Treasury bill portfolio at even the lower 5% withdrawal rate. But adding small amounts of equity quickly reduces the bankruptcy probability. With only a 0.5 stock proportion the probability of bankruptcy is already below 20% and reaches a minimum at a stock proportion of 0.8.

Our method of analysis could be used to help set the appropriate withdrawal rate. For example, the 20-year findings suggest 5% is too low a rate, because there is not much chance of bankruptcy for conservative portfolios. But 10% is too high since bankruptcy is certain for portfolios with substantial Treasury bill holdings. Further resampling can be performed for various withdrawal rates to help determine the best choice.

Conclusions

Our findings suggest that investors should use caution when considering high amounts of leverage. Despite the strong equity returns in the second half of the 20th century, maintaining highly leveraged portfolios over prolonged periods leads to declines in real wealth.

Even though current Canadian rules allow a 30% margin for most large stocks, we believe that investors should never reduce their margin below 50%. That is, a 200% stock allocation should be the maximum for even the most aggressive investors. This is especially important for a 30- or 40-year investment horizon, where higher leverage produces significant declines in ending wealth.

For the conservative investor, leverage may be unpalatable. If a risk-averse investor is concerned about underperforming a 100% Treasury bill portfolio, then any degree of leverage is unattractive, since the probability of falling below this benchmark steadily increases for stock proportions greater than 1.0.
Our findings also provide new perspectives on asset allocation during retirement years. Over some combinations of withdrawal rates and time horizons, a modest amount of leverage can reduce the chance of running out of money. Our results also confirm previous research findings that a Treasury bill portfolio may not provide a high enough return to meet the desired income stream.

References

Endnotes
1. Monthly data are reported by the Investment Dealers Association of Canada for outstanding debt in client margin accounts.
2. Most Canadian brokerage firms use the chartered banks’ prime rate as the benchmark for margin loans. In early August 2002, the Treasury bill yield was 2.5% and the prime rate was 4.5%. Individual investors may be charged a margin loan rate higher than the prime, such as 1% more.
3. Similarly, without rebalancing an unleveraged portfolio of stocks and Treasury bills, the stock proportion rises towards 100% as the stock value grows.
4. Figure 1 does not consider margin calls, which could reduce the most extreme losses by forcing rebalancing earlier in the month. Also, 400% is not possible under current margin regulations. But this is not a factor for more modest leverage amounts. The 150% stock portfolio would not have received a margin call anytime in the 52 years.
5. The MEL rule achieves maximum growth of wealth over the indefinite term. According to this rule, an individual should invest each period to maximize the expected value of the log of the single period gross return (see Markowitz (1976)).
6. The real stock returns have a monthly mean of 0.6447% and a standard deviation of 4.4895%, while the real T-bill returns have a 0.1636% mean and a 0.4619% standard deviation. These correspond to real annualized returns of 8.01% and 1.98% on stocks and T-bills, respectively.
7. The 1-year withdrawal period is not shown because bankruptcy is virtually impossible. We also omit the 40-year period as being an unusually long retirement horizon.
8. This problem was previously described in this journal by Ho, Milevsky, and Robinson (1994), and by the same authors in several subsequent studies.