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Asset Allocation and Risk

Using historic data and random variables to forecast returns

Historically, there has been a bias in the

risk premiums of equities compared to bonds—in favour of bonds. This is due, in part, to the poor performance of bond markets, where actual returns have fallen below expected levels from 1925 to 2001.

An expected real rate of return is now available in the yield on real return bonds, and it can be used in place of historic estimates. But we still have to look at the historic data for equities as the returns are so volatile. U.S. data from 1871 to 2001 indicates that there has been no trend in the real-equity rate of return. As a result, it is difficult to improve on this historic, arithmetic real rate of return, estimated at 8.9 per cent, as a forecast.

Canadian equity data indicates that historic real-equity returns have been about one per cent lower than this rate. This means that using the current expected return on the real bond with these real-equity returns, there is an arithmetic risk premium of four per cent to 5.3 per cent.

50/50 CHANCE

What does the future hold? If the equity return has an 18 per cent standard deviation next year, there is both a 50 per cent chance of getting more and less than the average real return of 8.9 per cent. More to the point, the probability of earning less than the 3.6 per cent real-bond return is about 40 per cent. This is the asset allocation risk for a one-year horizon. However, it is important to look at what happens over longer time periods.

As investment returns multiply, the overall return becomes skewed. For example, an investment with either a +100 per cent or -50 per cent return provides either \$200 or \$50 after one period for an average return of 25 per cent. However, after two periods, the possibili-

ties are a 25 per cent chance of \$400, a 50 per cent chance of \$100 and a 25 per cent chance of \$25. The average payoff is \$156.25 for a 25 per cent average return. In addition, there is now a 75 per cent chance of getting less than the average, since the average is pulled up by the slim chance of getting \$400.

Multiplying random variables together results in a specific, skewed distribution for prices known as the log-normal distribution. As a result, it's easiest to forecast by first estimating historic returns—assuming log-normality—and then using the log-normality assumption to forecast the probability of certain future events. For example, two critical events are the probability of earning the expected (historic average) equity return, as well as the probability, or risk, of failing to beat the bond market.

GOOD NEWS?

The outcome contains both good and bad news. First the bad news. The probability of earning the expected rate of return drops as the investment horizon lengthens. This is due to the increasingly skewed nature of long-run returns and the fact that the overall, multi-period return will skew towards the median, rather than the average. This means that pension funds should not base their investment returns on the basis of historic average returns, as they will probably be disappointed.

The good news is that the probability of beating the bond market goes up as time goes on. There is about a 40 per cent probability of failing to beat the bond market on a one-year basis, but this drops to about 20 per cent over 20 years. Risk does not go away as the asset allocation shifts more towards equities. However, for longer time periods, the risk does drop appreciably. ■