

Security Risk and Asset Allocation

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Estimating the Market Risk Premium

- ◆ Suppose we assume that the equity risk premium is constant (Ibbotson et al)

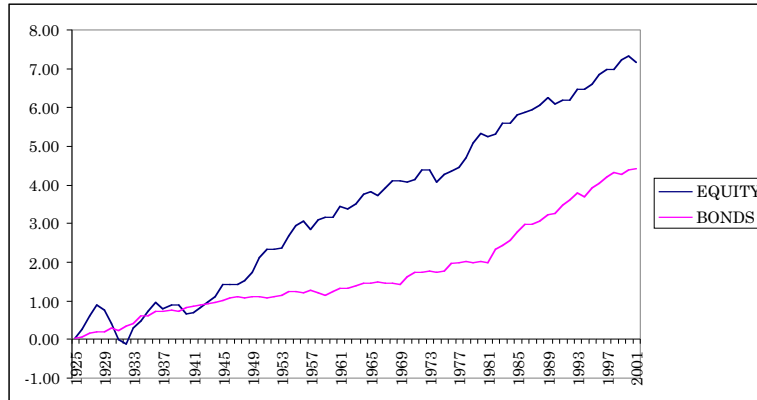
$$ERP = E(ERP) + \varepsilon_t$$

- ◆ have to minimize estimation risk
 - 2001: -12.57%
 - 2000: 7.51%
 - 1999: 31.71%
 - 1998: -1.58%
 - 1997: 14.98%
 - 1996: 28.35%

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Log-linear Equity and Bond Market Performance



Slope is $\ln(E_t/E_{t-1})$ which is continuously compounded the rate of return

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Annual Returns 1926-2001

	USA			CANADA		
	S&P500	Treasuries	Excess	TSE	Canadas	Excess
Arithmetic	12.65	5.67	6.98	11.51	6.34	5.17
Geometric	10.71	5.28	5.43	9.88	5.97	3.91
Least squares	11.19	4.53	6.66	10.51	5.20	5.31
St Dev	20.17	9.47		18.64	9.18	

Geometric Mean = Arithmetic Mean - .5*variance

For example, US variance is about 4%, so AM and GM diverge by about 2%

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US & Canadian Risk Premium

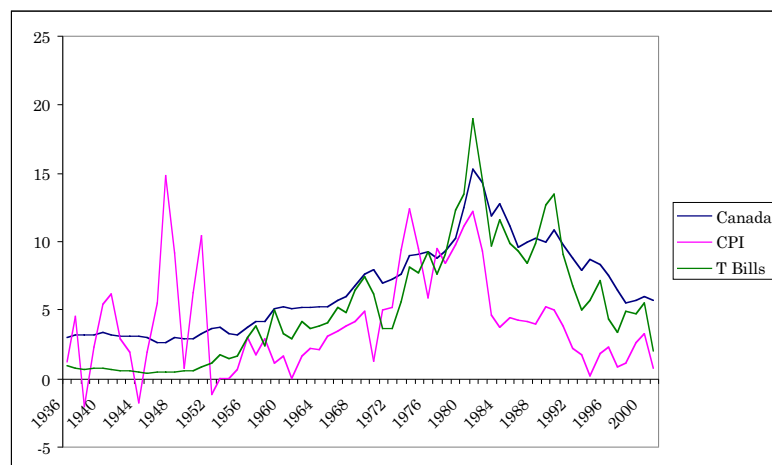
	Equity	Bonds	ERP
◆ Canada	11.51	6.34	5.17
◆ US	12.65	5.67	6.98
◆ Difference	+1.14	-0.67	1.81

*About two thirds one thirds equity bond markets
Standard error of the mean equity return is about
2.2%, so difference isn't statistically significant*

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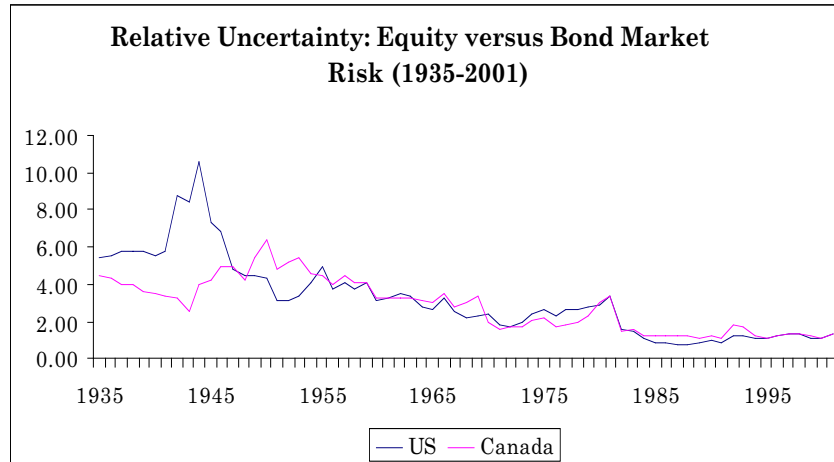


Bond Markets since 1936



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Ratio of 10-year rolling standard deviations of returns

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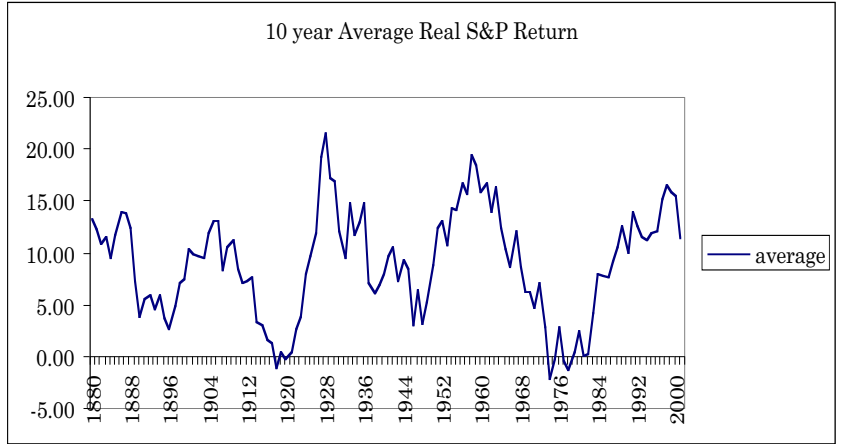


Conclusion

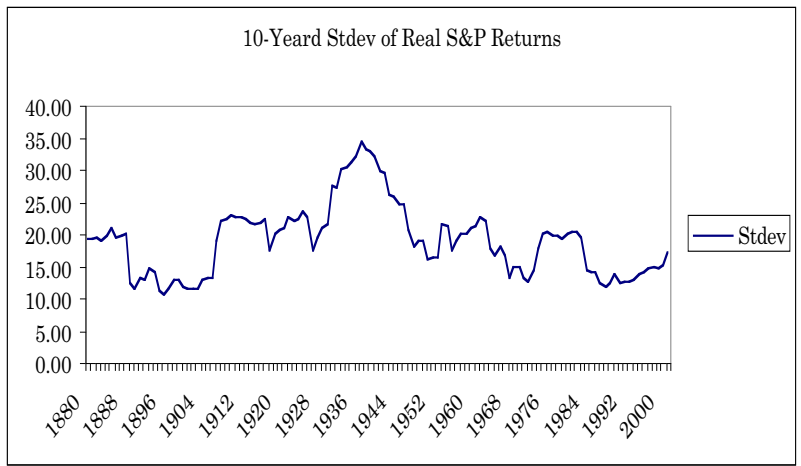
- ◆ **Equities outperform bonds**
 - About 5.0% in Canada
 - About 7.0% in US
- ◆ **Is Equity risk premium stable?**
 - Is equity return stable?
 - Bond return has not been stable!
 - Distinct periods
 - ❖ Up to early 1950's stable interest rates
 - ❖ Up to 1981 rising interest rates
 - ❖ Since 1981 falling interest rates
- ◆ **Use current yield**
 - Nominal yield: 5.9%
 - Real bond yield: 3.6%

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Implications

- ◆ Decline in US equity risk is only from the highs of 1920's-1930's
- ◆ Before & after equity risk was lower
- ◆ Average real equity return 8.89%
- ◆ Average annual standard deviation of 19.1%
- ◆ Canadian returns have been about 1.0% lower
- ◆ Given a real bond yield of 3.6% this implies an equity risk premium of 5.3% for US and 4.3% for Canada

- ◆ What does this mean for portfolio risk?

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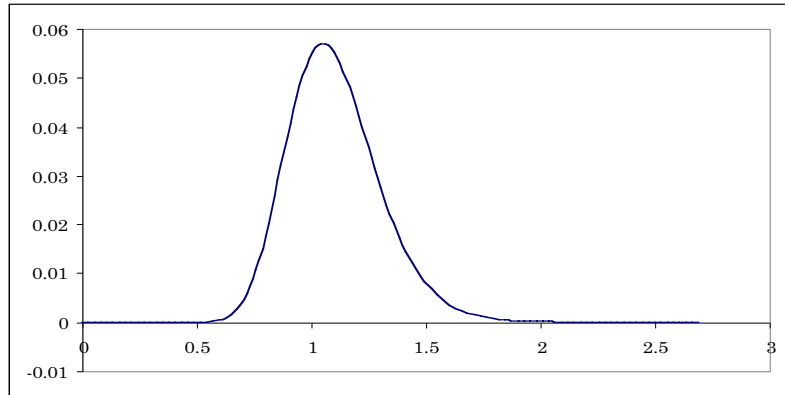
Risk

- ◆ Probability of incurring harm.
- ◆ Harm is losing money or failing to beat the bond market
- ◆ The risk in the equity market is the likelihood of failing to beat a 3.6% real return even though you expect to earn 8.9%

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One Period Investment



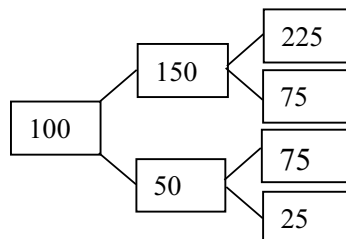
5.3/19% or 0.27 standard deviations below the mean or about 40% probability

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Implications

- ◆ Can't lose 100%: problems with assuming normality
- ◆ Skewed distribution of multi-period returns
- ◆ Slim possibility of very high payoffs
- ◆ Suppose payoff is +/- 50%



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Multiplicative Random Variables

- ◆ **Central limit theorem**
 - Additive Random variables: approximately a normal distribution
 - Multiplicative random variables: approximately a lognormal distribution (the logarithm of the price is normally distributed)

- ◆ **Lognormal distribution**
 - Wealth can not be negative
 - Limiting distribution for all multi-period investments

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Lognormal Distribution

- ◆ **Equity securities**

$$\text{Ln}(W_T) \sim \text{Ln}(W_0 + N((r - .5 * \sigma^2)T, \sigma \sqrt{T}))$$

$$\text{Ln}(W_T) \sim N(.0713 T, .18\sqrt{T})$$

- ◆ **Real Bond**

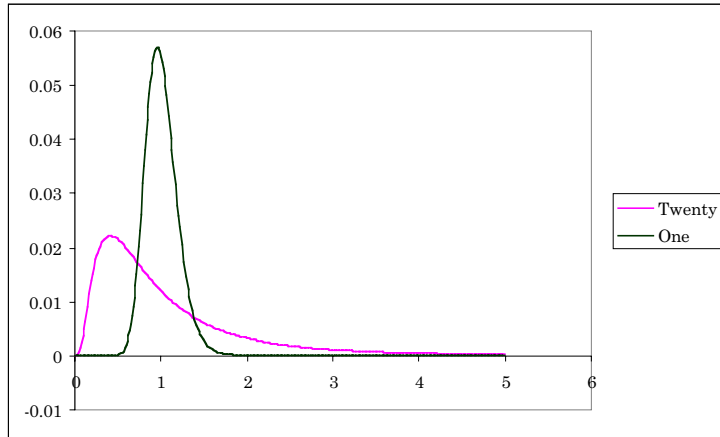
$$\text{Ln}(W_T) \sim \text{Ln}(W_0) + T\text{Ln}(1.036)$$

$$\text{Ln}(W_T) \sim 0.036T$$

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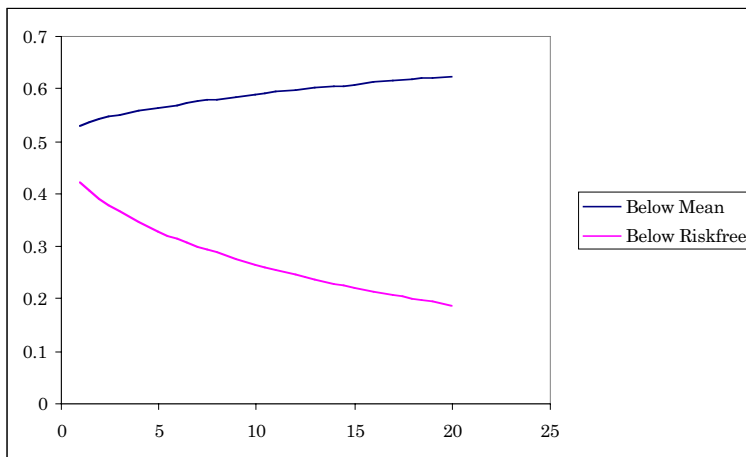
Multi-period



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Multi-period Wealth



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Over Time

- ◆ Distribution of wealth becomes skewed
- ◆ Likelihood of earning the expected return goes down

Do not plan on the basis of historic earned risk premia!

- ◆ Likelihood of outperforming the real bond increases

Equities are less risky in the long run, but can still earn less than a risk free real bond!

